Q2

a. $a_k = \begin{cases} jk; & |k| < 3 \\ 0 & \text{otherwise} \end{cases}$

$$x(t) = -2\sin(\pi t/2) - 4\sin(\pi t) \qquad \text{for } 0 \le t < 4.$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt} = -2je^{-j\frac{2\pi}{4}2t} - je^{-j\frac{2\pi}{4}t} + je^{j\frac{2\pi}{4}t} + 2je^{j\frac{2\pi}{4}2t}$$

$$= -2\sin(\pi t/2) - 4\sin(\pi t)$$

b. $b_k = \begin{cases} 1; & k \text{ odd} \\ 0; & k \text{ even} \end{cases}$

$$x(t) = 2\delta(t) - 2\delta(t-2) \qquad \text{for } 0 \le t < 4.$$

$$x(t) = \sum_{k=-\infty}^{\infty} b_k e^{j\frac{2\pi}{T}kt} = \sum_{\substack{k=-\infty \\ k \text{ odd}}}^{\infty} e^{j\pi kt/2}$$

Unfortunately, this sum is not easy to close. However, it is closely related to the synthesis formula for an impulse train,

$$x_\delta(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) = \sum_{k=-\infty}^{\infty} a_k e^{j\pi kt/2} = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{j\pi kt/2}.$$

If there were two impulses per period instead of one, then

$$x_{2\delta}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) + \delta(t - \frac{T}{2} - kT) = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{j2\pi kt/4} + \frac{1}{T} e^{j2\pi k(t-2)/4}$$

$$= \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{j\pi kt/2} \left(1 + e^{j\pi k}\right) = \sum_{\substack{k=-\infty \\ k \text{ even}}}^{\infty} \frac{2}{T} e^{j\pi kt/2}$$

It follows that $x(t) = Tx_\delta(t) - \frac{T}{2}x_{2\delta}(t) = 4x_\delta(t) - 2x_{2\delta}(t)$ so that $x(t)$ is an alternating sequence of impulses

$$x(t) = \sum_{l=-\infty}^{\infty} 2\delta(t - 4l) - 2\delta(t - 2 - 4l)$$

Q3

```
[h1,f] =
freqz(d1,1024,fs);
[h2,~] =
freqz(d2,1024,fs);
[h3,~] =
freqz(d3,1024,fs);

plot(f,mag2db(abs(
[h1 h2 h3])))
legend('Steepness
= 0.5','Steepness =
0.8','Steepness =
0.95',
```